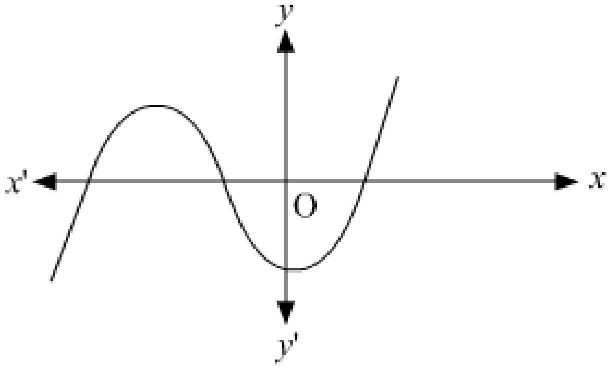
Maths (Basic) Delhi (Set 2)

General Instructions :
(i) This question paper comprises four sections – A, B, C and D. This question paper carries 40 questions. All questions are compulsory:
(ii) Section A : Q. No. 1 to 20 comprises of 20 questions of one mark each.
(iii) Section B : Q. No. 21 to 26 comprises of 6 questions of two marks each.
(iv) Section C: Q. No. 27 to 34 comprises of 8 questions of three marks each.
(v) Section D : Q. No. 35 to 40 comprises of 6 questions of four marks each.
(vi) There is no overall choice in the question paper. However, an internal choice has been provided in 2 questions of one mark each, 2 questions of two marks each, 3 questions of three marks each and 3 questions of four marks each. You have to **attempt only one of the choices** in such questions.
(vii) In addition to this, separate instructions are given with each section and question, wherever necessary.
(viii) Use of calculators is not permitted.

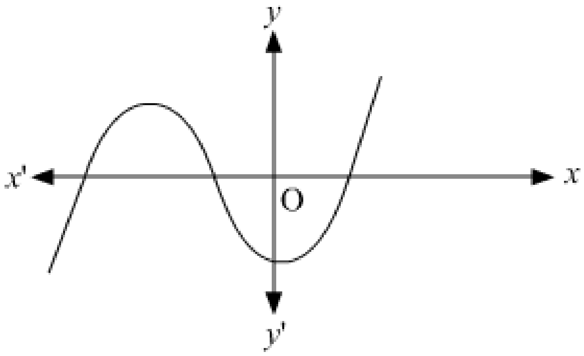
Question 1

The graph of a polynomial is shown in figure, then the number of its zeroes is



(a) 3
(b) 1
(c) 2
(d) 4

Solution:



The number of zeroes is 3 as the graph of the polynomial cuts the x-axis at 3 points which means the value of y is zero at those points.

Hence, the correct answer is option (a).

Question 2

225 can be expressed as
(a) 5×3^2
(b) $5^2 \times 3$
(c) $5^2 \times 3^2$
(d) $5^3 \times 3$

Solution:

225 can be written as:
$225 = 3 \times 3 \times 5 \times 5 = 3^2 \times 5^2$
Hence, the correct answer is option (c).

Question 3

The probability that a number selected at random from the numbers 1, 2, 3, ..., 15 is a multiple of 4 is
(a) 4/15
(b) 2/15
(c) 1/15
(d) 1/5

Solution:

The multiples of 4 from 1 to 15 are 4, 8 and 12.
Hence, the probability of selecting a multiple of four $= \frac{3}{15} = \frac{1}{5}$
Hence, the correct answer is option (c).

Question 4

$2.\overline{35}$ is

(a) an integer
(b) a rational number
(c) an irrational number
(d) a natural number

Solution:

$$2.\overline{35} = 2.35353535...$$

$2.\overline{35}$ is a non-terminating repeating decimal.

And we know that every non-terminating repeating decimal is a rational number.
Hence, the correct answer is option (b).

Question 5

The median and mode respectively of a frequency distribution are 26 and 29. Then its mean is
(a) 27.5
(b) 24.5
(c) 28.4
(d) 25.8

Solution:

The empirical relationship between mean, median and mode is

$$\text{Mode} = 3\text{Median} - 2\,\text{Mean}$$
$$\Rightarrow 2\text{Mean} = 3\text{Median} - \text{Mode}$$
$$\Rightarrow 2\text{Mean} = 3 \times 26 - 29$$
$$\Rightarrow \text{Mean} = \frac{49}{2} = 24.5$$

Hence, the correct answer is option (b)

Question 6

HCF of 144 and 198 is
(a) 9
(b) 18
(c) 6
(d) 12

Solution:

Using Euclid's division algorithm,

$$198 = 144 \times 1 + 54$$
$$144 = 54 \times 2 + 36$$
$$54 = 36 \times 1 + 18$$
$$36 = 18 \times 2 + 0$$

⇒HCF of 144 and 198 is 18.

Hence, the correct answer is option (b).

Question 7

If the distance between the points A(4, *p*) and B(1, 0) is 5 units, then the value(s) of *p* is (are)

(a) 4 only
(b) −4 only
(c) ±4
(d) 0

Solution:

The distance between the points A(4, p) and B(1, 0) is given by

$$\sqrt{(4-1)^2 + (p-0)^2}$$
$$= \sqrt{3^2 + p^2}$$
$$= \sqrt{9 + p^2}$$

According to the question,

$$\sqrt{9+p^2} = 5$$
$$\Rightarrow 9 + p^2 = 5^2$$
$$\Rightarrow p^2 = 25 - 9$$
$$\Rightarrow p = \pm\sqrt{16}$$
$$\Rightarrow p = \pm 4$$

Hence, the required answer is option (c).

Question 8

The area of a triangle with vertices A(5, 0), B(8, 0) and C(8, 4) in square units is

(a) 20
(b) 12
(c) 6
(d) 16

Solution:

Area of a triangle with vertices $\mathrm{A}(x_1, y_1), \mathrm{B}(x_2, y_2), \mathrm{C}(x_3, y_3)$ is $\frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$
Now, Area of a triangle with vertices A(5, 0), B(8, 0), C(8, 4) is
$=\frac{1}{2}|5(0-4)+8(4-0)+8(0-0)|$
$=\frac{1}{2}|-20+32+0|$
$=\frac{1}{2}|12|$
$=\frac{1}{2}\times 12=6$
Hence, the correct answer is option (c).

Question 9

The sum and product of the zeroes of a quadratic polynomial are 3 and – 10 respectively. The quadratic polynomial is
(a) $x^2 - 3x + 10$
(b) $x^2 + 3x - 10$
(c) $x^2 - 3x - 10$
(d) $x^2 + 3x + 10$

Solution:

Let the zeroes of the required polynomial be αα and ββ respectively.
So, according to the question, we have

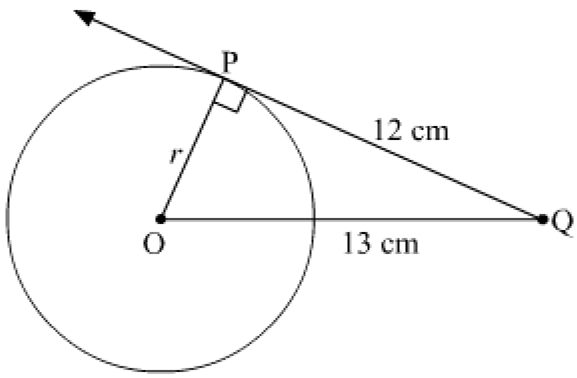
$\alpha+\beta=3$ and $\alpha\beta=-10$.
Hence, the required quadratic polynomial is $x^2-(\alpha+\beta)x+\alpha\beta=x^2-3x-10$
Hence, the correct answer is option (c).

Question 10

From an external point Q, the length of tangent to a circle is 12 cm and the distance of Q from the centre of circle is 13 cm. The radius of circle (in cm) is
(a) 10
(b) 5
(c) 12
(d) 7

Solution:

Let *r* be the radius of the circle.



We know that the radius is always perpendicular to the tangent at their point of contact.
Using Pythagoras theorem in ΔPOQ, we have

$OQ^2 = OP^2 + PQ^2$
$\Rightarrow 13^2 = r^2 + 12^2$
$\Rightarrow 169 = r^2 + 144$
$\Rightarrow r^2 = 25$
$\Rightarrow r = 5$

Hence, the correct answer is option (b).

Question 11

Fill in the blank.
If tan (A + B) = $\sqrt{3}$ and tan (A − B) = $\frac{1}{\sqrt{3}}$, A > B, then the value of A is __________.

Solution:

Given, $\tan(A+B)=\sqrt{3}$ and $\tan(A-B)=\frac{1}{\sqrt{3}}$
Therefore, $A+B=60^\circ$ and $A-B=30^\circ$
Adding the two equations, we get
$2A=90^\circ$
$\Rightarrow A=\frac{90^\circ}{2}=45^\circ$

Question 12

Fill in the blank.
The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of the first triangle is 9 cm, then the corresponding side of second triangle is ___________.

Solution:

We know that, the ratio of the perimeter of two similar triangles = ratio of their corresponding sides

$\Rightarrow \frac{25}{15} = \frac{9}{x}$

$\Rightarrow x = \frac{9 \times 15}{25} = \frac{27}{5}$

$\Rightarrow x = 5.4$

Hence, the corresponding side of the second triangle is 5.4 cm.

Question 13

Fill in the blank.

If the equations $kx - 2y = 3$ and $3x + y = 5$ represent two intersecting lines at unique point, then the value of k is __________.

OR

Fill in the blank.

If quadratic equation $3x^2 - 4x + k = 0$ has equal roots, then the value of k is

____________.

Solution:

Since the given equations represent two lines intersecting at a unique point, they've got a unique solution.

Therefore,

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{k}{3} \neq \frac{-2}{1}$

$\Rightarrow k \neq -6$

Hence, the given pair of linear equations in two variables will have a unique solution for all values of k except −6.

OR

Given that the quadratic equation $3x^2 - 4x + k$ has equal roots.

$\Rightarrow D = 0$

$\Rightarrow b^2 - 4ac = 0$

where $a = 3,\ b = -4,\ c = k$

$\Rightarrow (-4)^2 - 4 \times 3 \times k = 0$

$\Rightarrow 16 - 12k = 0$

$\Rightarrow 12k = 16$

$\Rightarrow k = \frac{4}{3}$

$\Rightarrow$ The value of k is $\frac{4}{3}$

Question 14

Fill in the blank.
If the point C(k, 4) divides the line segment joining two points A(2, 6) and B(5, 1) in ratio 2 : 3, the value of k is ___________.

OR

Fill in the blank.
If points A(−3, 12), B(7, 6) and C(x, 9) are collinear, then the value of x is __________.

Solution:

Using the Section formula, we have

$k = \frac{2 \times 5 + 3 \times 2}{2 + 3}$

$\Rightarrow k = \frac{10 + 6}{5}$

$\Rightarrow k = 3.2$

OR

For collinear points, Area = 0.
i.e.

$\frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right| = 0$

$\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right| = 0$

$\left|-3(6 - 9) + 7(9 - 12) + x(12 - 6)\right| = 0$

$\left|-3(-3) + 7(-3) + x(6)\right| = 0$

$\left|9 - 21 + 6x\right| = 0$

$\left|6x - 12\right| = 0$

$\Rightarrow 6x - 12 = 0$

$\Rightarrow 6x = 12$

$\Rightarrow x = 2$

The value of x is 2.

Question 15

Fill in the blank.
The value of $\sin^2 65° + \sin^2 25°$ is

Solution:

The given expression can be simplified as

$$\sin^2 65^\circ + \sin^2 25^\circ$$
$$= \sin^2 65^\circ + \sin^2 (90^\circ - 65^\circ)$$
$$= \sin^2 65^\circ + \cos^2 65^\circ \qquad [\because \sin (90^\circ - \theta) = \cos\theta]$$
$$= 1 \qquad (\because \sin^2\theta + \cos^2\theta = 1)$$

Question 16

The n^{th} term of an AP is $(7 - 4n)$, then what is its common difference?

Solution:

Given: n^{th} term of an AP is $(7-4n)$

Since n^{th} term of an AP is given by

$$T_{\text{n}} = a + (n-1)d, \text{ where } a = \text{First Term}, d = \text{Common Difference}$$

Therefore, we have

$$7 - 4n = a + (n-1)d$$
$$\Rightarrow 7 - 4n = (a - d) + nd$$

Comparing both sides, we get

$$d = -4$$

Hence, the common difference is -4.

Question 17

If a pair of dice is thrown once, then what is the probability of getting a sum of 8?

Solution:

Total number of outcomes when two dices are thrown simultaneously is given by,

$$\text{Total outcomes} = 6 \times 6 = 36$$

Favorable pair of outcomes for getting a sum of 8 is given by,

$$(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)$$
$$\Rightarrow \text{Total number of favourable outcomes} = 5$$

Therefore,

$$\text{Required probability} = \frac{\text{Total number of favourable outcomes}}{\text{Total outcomes}}$$
$$= \frac{5}{36}$$

Question 18

The areas of two circles are in the ratio 9 : 4, then what is the ratio of their circumferences?

Solution:

Given: Ratio of the areas of the two circle is 9 : 4.
Let the areas of the the two circles be A_1 and A_2.
Hence, $\frac{A_1}{A_2} = \frac{9}{4}$ .....(1)
Let the radii of the two circles be r_1 and r_2.

$\frac{\text{Area of the first circle}}{\text{Area of the second circle}} = \frac{\pi {r_1}^2}{\pi {r_2}^2}$

$\Rightarrow \frac{\pi {r_1}^2}{\pi {r_2}^2} = \frac{9}{4}$ [From (1)]

$\Rightarrow \frac{{r_1}^2}{{r_2}^2} = \frac{9}{4}$

$\Rightarrow \frac{r_1}{r_2} = \frac{3}{2}$ .....(2)

Let the circumferences of the two circles be C_1 and C_2.

$\frac{C_1}{C_2} = \frac{2\pi r_1}{2\pi r_2}$

$\Rightarrow \frac{C_1}{C_2} = \frac{r_1}{r_2}$

$\Rightarrow \frac{C_1}{C_2} = \frac{3}{2}$ [From (2)]

The ratio of their circumference is 3 : 2.

Question 19

If 5tanθ = 3, then what is the value of $\left(\frac{5 \sin \theta - 3 \cos \theta}{4 \sin \theta + 3 \cos \theta}\right)$?

Solution:

Given that $5 \tan \theta = 3$

$\Rightarrow \tan \theta = \frac{3}{5}$

$\Rightarrow \frac{\sin \theta}{\cos \theta} = \frac{3}{5}$

Let $\sin \theta = 3k$ and $\cos \theta = 5k$, where k is any integer.

Consider the given expression:

$\frac{5 \sin \theta - 3 \cos \theta}{4 \sin \theta + 3 \cos \theta}$

$= \frac{5(3k) - 3(5k)}{4(3k) + 3(5k)}$

$= \frac{0}{27k}$

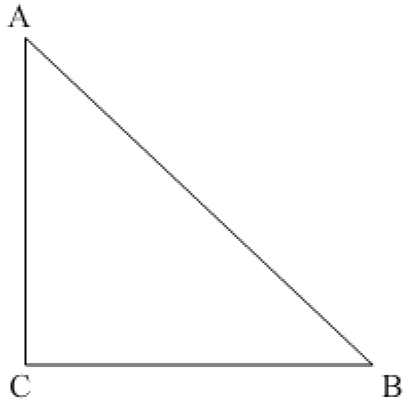
$= 0$

Question 20

ΔABC is isosceles with AC = BC. If $AB^2 = 2AC^2$, then find the measure of $\angle$C.

Solution:

Given that the triangle ABC is isosceles such that AC = BC.



Also, $AB^2 = 2\,AC^2$.

$$\Rightarrow AB^2 = AC^2 + AC^2$$
$$\Rightarrow AB^2 = AC^2 + BC^2 \qquad (\because AC = BC)$$

Therefore, by the converse of the Pythagoras Theorem, we can say that ABC is a right-angled triangle.

Hence, $\angle C = 90°$

Question 21

Prove that $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta.$

Prove that $\frac{\tan^2\ \theta}{1+\tan^2\ \theta} + \frac{\cot^2\ \theta}{1+\cot^2\ \theta} = 1$

Solution:

Consider the LHS:

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$
$$= \sqrt{\frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}}$$
$$= \sqrt{\frac{(1-\sin\theta)^2}{1^2-\sin^2\theta}}$$

$$= \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$$
$$= \frac{1-\sin\theta}{\cos\theta}$$
$$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$
$$= \sec\theta - \tan\theta$$

=RHS

Hence proved.

OR

Consider the LHS:

$$\frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\cot^2\theta}{1+\cot^2\theta}$$
$$= \frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\frac{1}{\tan^2\theta}}{1+\frac{1}{\tan^2\theta}}$$
$$= \frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\frac{1}{\tan^2\theta}}{\frac{1+\tan^2\theta}{\tan^2\theta}}$$
$$= \frac{\tan^2\theta}{1+\tan^2\theta} + \frac{1}{1+\tan^2\theta}$$
$$= \frac{1+\tan^2\theta}{1+\tan^2\theta}$$
$$= 1$$
=RHS

Question 22

Two different dice are thrown together, find the probability that the sum a of the numbers appeared is less than 5.

OR

Find the probability that 5 Sundays occur in the month of November of a randomly selected year.

Solution:

Two dices are thrown simultaneously.

So, the total number of outcomes will be 6^2 = 36.

Now, all the favorable pairs whose sum is less than 5 is given by (1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1).

The total number of favorable outcomes is 6.

Hence, the required probability = 6/36 = 1/6.

OR

In any randomly selected year, the Month of November will have 30 days.

Now out of these 30 days, we will have 4 complete weeks (i.e. 28 days) having 4 Sundays.

For the remaining two days, we have the following possibilities:
(i) Saturday and Sunday,
(ii) Sunday and Monday,
(iii) Monday and Tuesday,
(iv) Tuesday and Wednesday,
(v) Wednesday and Thursday,
(vi) Thursday and Friday,
(vii) Friday and Saturday.

Thus, the possibility of having a 5th Sunday = 2/7.

Question 23

A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball at random from the bag is three times that of a red ball, find the number of blue balls in the bag.

Solution:

Given, the number of red balls in the bag = 5

Let the number of blue balls in the bag = x

Therefore, the total number of balls in the bag = $x + 5$

Let R and B denote the events of drawing a red ball and a blue ball respectively from the bag.

Then, according to the question, we have

$P(B) = 3P(R)$

$\Rightarrow \frac{x}{x+5} = 3 \times \frac{5}{x+5}$

$\Rightarrow \frac{x}{x+5} = \frac{15}{x+5}$

$\Rightarrow x = 15$

Hence, the number of blue balls in the bag is 15.

Question 24

Divide the polynomial $(9x^2 + 12x + 10)$ by $(3x + 2)$ and write the quotient and the remainder.

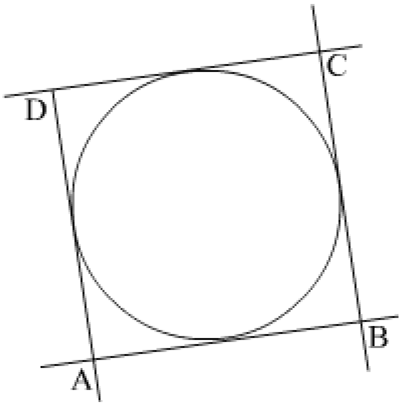
Divide the two polynomials using the long division method.

$$\begin{array}{r|l} & 3x + 2 \\ \hline 3x + 2 & 9x^2 + 12x + 10 \\ & \underline{-9x^2 - 6x} \\ & 6x + 10 \\ & \underline{-6x - 4} \\ & 6 \end{array}$$

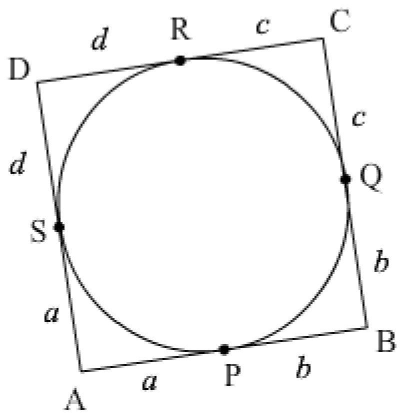
Hence, the quotient obtained is $(3x + 2)$ and the remainder is 6.

Question 25

In the given figure a circle touches all the four sides of a quadrilateral ABCD in which AB = 6 cm, BC = 7 cm and CD = 4 cm. Find AD.



The figure given in the question is below.



From the property of tangents we know that, the length of two tangents drawn from the same external point will be equal. Therefore we have the following:

SA = AP, PB = BQ, QC = CR, DR = DS.

For our convenience, let us represent SA and AP by a, PB and BQ by b, QC and CR by c and DR and DS by d.

It is given in the problem that, AB = 6

By looking at the figure we can rewrite the above equation as follows:
AP + PB = 6
$\Rightarrow a + b = 6$
$\Rightarrow b = 6 - a$

Similarly we have, BC = 7
$\Rightarrow$ BQ + QC = 7
$\Rightarrow b + c = 7$
$\Rightarrow 6 - a + c = 7$
$\Rightarrow c = a + 1$

Also, CD = 4
$\Rightarrow$ CR + RD = 4
$\Rightarrow c + d = 4$
$\Rightarrow a + 1 + d = 4$
$\Rightarrow a + d = 3$

As per our representations in the previous section, we can write the above equation as follows:

SA + DS = 3

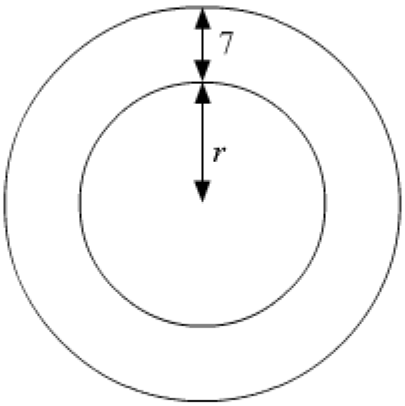
By looking at the figure, we have AD = 3.

Thus we have found that the length of side AD is 3 cm.

Question 26

A road which is 7 m wide surrounds a circular park whose circumference is 88 m. Find the area of the road.

Solution:



Let the radius of the circular park be *r*.
Its circumference is given to be 88 m.

$$\Rightarrow 2\pi r = 88$$
$$\Rightarrow r = 88 \times \frac{1}{2} \times \frac{7}{22} = 14 \text{ m}$$

The road is given to be 7 m wide.
Therefore, the area of the road = Area of outer boundary of road — Area of the park

$$= \pi(r+7)^2 - \pi r^2$$
$$= \pi(14+7)^2 - \pi(14)^2$$
$$= \pi\left(21^2 - 14^2\right)$$
$$= \frac{22}{7} \times 245$$
$$= 770 \text{ m}^2$$

Question 27

Draw a circle of radius 4 cm. From a point 7 cm away from the centre of circle. Construct a pair of tangents to the circle.

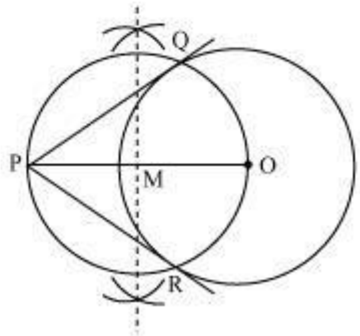
OR

Draw a line segment of 6 cm and divide it in the ratio 3 : 2.

Solution:

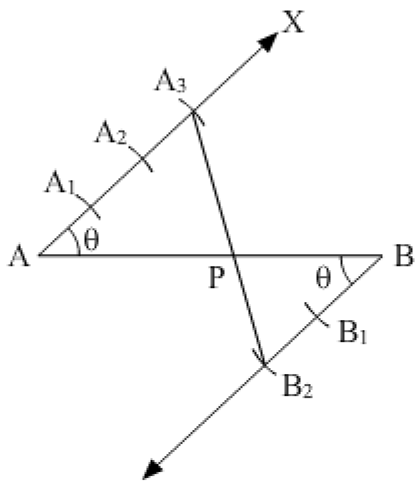
A pair of tangents to the given circle can be constructed as follows.

Step 1 Draw a circle of 4 cm radius and O as centre. Locate a point P, 7 cm away from O. Join OP.
Step 2 Bisect OP. Let M be the mid-point of PO.
Step 3 Taking M as centre and MO as radius, draw a circle.
Step 4 Let this circle intersect the previous circle at point Q and R.
Step 5 Join PQ and PR. PQ and PR are the required tangents.



OR

We need to follow the following steps to construct the given



Step of construction
Step I- First of all we draw a line segment AB = 6 cm.

Step II- We draw a ray AX making an acute angle with AB.
Step III- Draw a ray BY parallel to AX by making an acute angle ∠ABY = ∠BAX.

Step IV- Mark three points A_1, A_2, A_3 on AX and two points B_1, B_2 on BY in such a way that $AA_1 = A_1A_2 = A_2A_3 = BB_1 = B_1B_2$.

Step V- Join A_3B_2. This line intersects AB at a point P.

Thus, the given line segment AB has been divided internally in the ratio of 3 : 2

Question 28

Prove that (1 + tan A − sec A) × (1 + tan A + sec A) = 2 tan A

OR

Prove that $\frac{\operatorname{cosec}\ \theta}{\operatorname{cosec}\ \theta-1}+\frac{\operatorname{cosec}\ \theta}{\operatorname{cosec}\ \theta+1}=2\sec^2\theta$

Solution:

Consider the LHS:

$$(1+\tan A-\sec A)(1+\tan A+\sec A)$$
$$=[(1+\tan A)-\sec A][(1+\tan A)+\sec A]$$
$$=(1+\tan A)^2-\sec^2 A$$
$$=1+2\tan A+\tan^2 A-\sec^2 A$$
$$=1+2\tan A+(-1)$$
$$=2\tan A$$

=RHS

Hence proved.

OR

LHS

$$=\frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta-1}+\frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta+1}$$
$$=\frac{\operatorname{cosec}\theta(\operatorname{cosec}\theta+1)+\operatorname{cosec}\theta(\operatorname{cosec}\theta-1)}{\operatorname{cosec}^2\theta-1}$$
$$=\frac{\operatorname{cosec}^2\theta+\operatorname{cosec}\theta+\operatorname{cosec}^2\theta-\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta-1}$$
$$=\frac{2\operatorname{cosec}^2\theta}{\cot^2\theta}\quad(\because \operatorname{cosec}^2\theta-1=\cot^2\theta)$$
$$=\frac{2}{\sin^2\theta}\times\frac{\sin^2\theta}{\cos^2\theta}\quad\left(\because \operatorname{cosec}^2\theta=\frac{1}{\sin^2\theta}\ \text{ and }\ \cot^2\theta=\frac{\cos^2\theta}{\sin^2\theta}\right)$$
$$=\frac{2}{\cos^2\theta}$$
$$=2\sec^2\theta\ \left(\because\ \sec^2\theta=\frac{1}{\cos^2\theta}\right)$$
$$=\text{RHS}$$

Hence proved.

Question 29

Given that √3 is an irrational number, show that (5 + 2√3) is an irrational number.

OR

An army contingent of 612 members is to march behind an army band of 48 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Solution:

Given: $\sqrt{3}$ is an irrational number.
To prove: $5+2\sqrt{3}$ is an irrational number.
Proof:
Suppose $5+2\sqrt{3}$ is a rational number.
Therefore it can be written in $\frac{p}{q}$ form, where p and q are coprime integers.

$\Rightarrow 5+2\sqrt{3}=\frac{p}{q}$

$\Rightarrow 2\sqrt{3}=\frac{p}{q}-5=\frac{p-5q}{q}$

$\Rightarrow \sqrt{3}=\frac{p-5q}{2q}$

Since p and q are integers, therefore $\frac{p-5q}{2q}$ must be a rational number.
But this is a contradiction as the LHS is an irrational number.
Our supposition was wrong.
Hence, $5+2\sqrt{3}$ is an irrational number.
Hence proved.

OR

We are given that an army contingent of 612 members is to march behind an army band of 48 members in a parade. The two groups are to march in the same number of columns. We need to find the maximum number of columns in which they can march.

Members in army = 612
Members in band = 48.
Therefore,

The maximum number of columns = H.C.F of 612 and 48.
By applying Euclid's division lemma

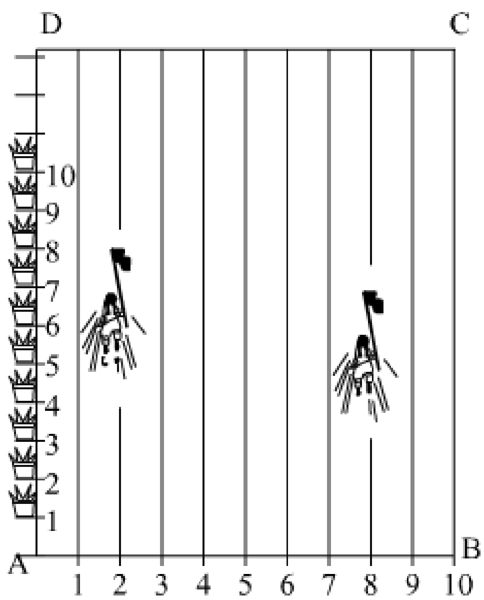
$612=48\times 12+36$
$48=36\times 1+12$
$36=12\times 3+0$

Therefore, H.C.F. = 12
Hence, the maximum number of columns in which they can march is 12.

Question 30

Read the following passage carefully and then answer the questions given at the end.

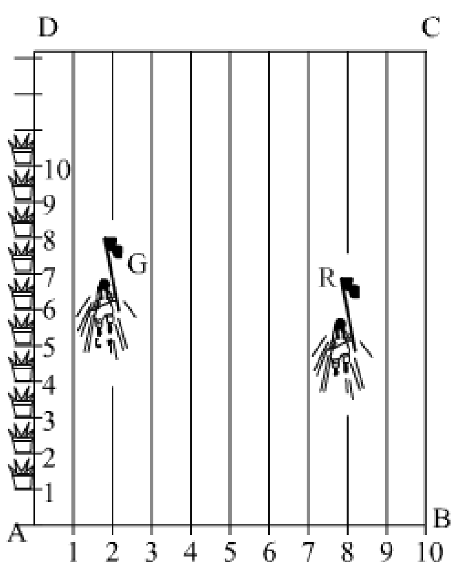
To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in Figure. Niharika runs 1/4th the distance AD on the 2nd line and posts a green flag. Preet runs 1/5th the distance AD on the eighth line and posts a red flag.



(i) What is the distance between the two flags?

(ii) If Rashmi has to post a blue flag exactly half way between the line segment joining the two flags, where should she post the blue flag?

Solution:



It can be observed that Niharika posted the green flag at $\frac{1}{4}$ of the distance AD i.e., $\left(\frac{1}{4}\times 100\right)\text{m} = 25\text{m}$ from the starting point of 2nd line. Therefore, the coordinates of this point G is (2, 25).

Similarly, Preet posted red flag at $\frac{1}{5}$ of the distance AD i.e., $\left(\frac{1}{5}\times 100\right)\text{m} = 20\text{ m}$ from the starting point of 8th line. Therefore, the coordinates of this point R are (8, 20).

(i) Distance between these flags by using distance formula = GR

$= \sqrt{(8-2)^2 + (25-20)^2} = \sqrt{36+25} = \sqrt{61}\text{ m}$

(ii) The point at which Rashmi should post her blue flag is the mid-point of the line joining these points. Let this point be A (*x*, *y*).

$x = \frac{2+8}{2}, \quad y = \frac{25+20}{2}$

$x = \frac{10}{2} = 5, \ y = \frac{45}{2} = 22.5$

Hence, $\text{A}(x, y) = (5, 22.5)$

Therefore, Rashmi should post her blue flag at 22.5m on 5th line.

Question 31

Solve graphically: $2x + 3y = 2$, $x - 2y = 8$

Solution:

Given:

$2x + 3y = 2$

$x - 2y = 8$

Consider the first equation $2x + 3y = 2$.

x	4	1
y	−2	0

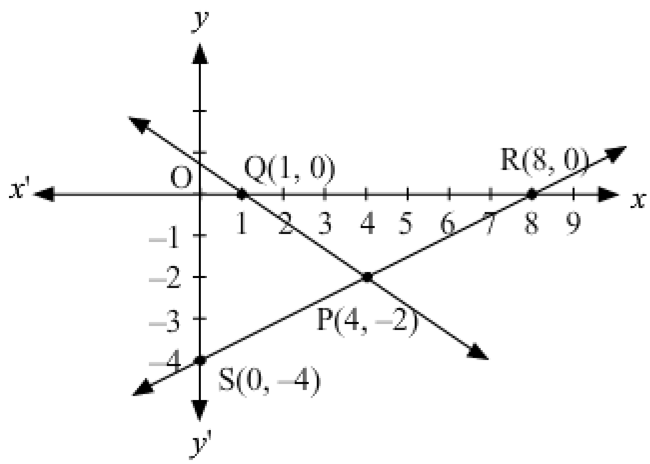
Hence, the points are $\text{P}(4, \ -2)$ and $\text{Q}(1, \ 0)$ respectively.

Considering the second equation $x - 2y = 8$.

x	0	8
y	−4	0

Hence, the points are $\text{S}(0, \ -4)$ and $\text{R}(8, \ 0)$ respectively.

Now drawing the two lines on the coordinate axis, we get



Hence, the point of intersection of these two line is $P(4, \ -2)$.

Question 32

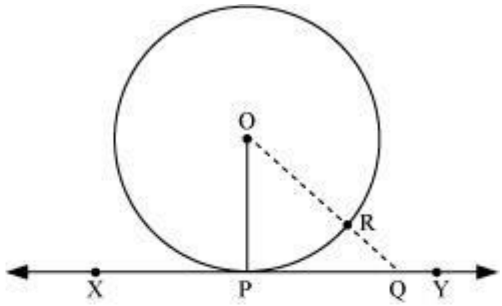
Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

Solution:

Given: A circle with centre O; XY is a tangent to the circle at point P.
To prove: OP ⊥ XY

Proof:

Let Q be a point on XY. Join OQ intersecting the circle at point R.



Then, OR < OQ
However, OP = OR [Radii]
∴ OP < OQ

Since Q was an arbitrary point on XY, OP is shorter than any other line segment joining O to any point of XY, other than P.

In other words, OP is the shortest distance between the point O and the line XY.

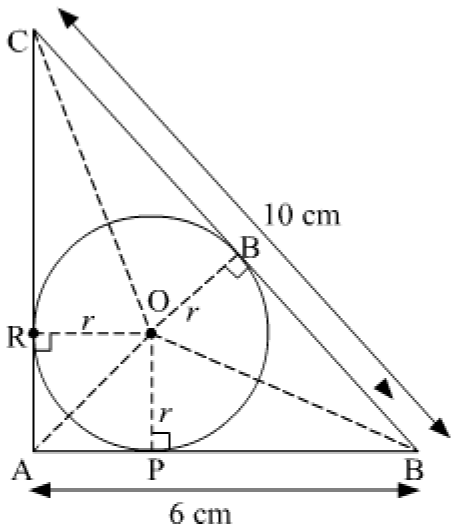
However, the shortest distance between a point and a line is the perpendicular distance.

Thus, OP ⊥ XY

Question 33

A right triangle ABC, right angled at A, is circumscribing a circle. If AB = 6 cm and BC = 10 cm, find the radius of the circle.

Solution:



Suppose the center of the circle is O and the sides AB, BC and AC of ΔABC be tangents to the circle at points P, Q and R respectively.

Now, we know that the tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore\ OP \perp AB,\ OQ \perp BC\ \text{and}\ OR \perp AC$

Also, $\triangle ABC$ is a right angled triangle

$\therefore\ BC^2 = AB^2 + AC^2$

$\Rightarrow AC = \sqrt{BC^2 - AB^2} = \sqrt{10^2 - 6^2} = 8\ \text{cm}$

Now, $ar(\triangle ABC) = ar(\triangle AOB) + ar(\triangle BOC) + ar(\triangle AOC)$

$\Rightarrow \frac{1}{2} \times AB \times AC = \frac{1}{2} \times AB \times OP + \frac{1}{2} \times BC \times OQ + \frac{1}{2} \times AC \times OR$

$\Rightarrow \frac{1}{2} \times 6 \times 8 = \frac{1}{2} \times 6 \times r + \frac{1}{2} \times 10 \times r + \frac{1}{2} \times 8 \times r$
$\Rightarrow 24 = \frac{1}{2} \times r \times (6 + 10 + 8)$
$\Rightarrow 24 = 12r$
$\Rightarrow r = 2 \text{ cm}$
Hence the radius of the circle is 2 cm.

Question 34

Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$, and verify the relationship between the zeroes and the coefficients.

Solution:

Given polynomial is
$x^2 + 7x + 10$
$= x^2 + 2x + 5x + 10 \qquad \text{(splitting the middle term)}$
$= x(x+2) + 5(x+2)$
$= (x+2)(x+5)$

Therefore, the zeroes of the given quadratic polynomial are $\alpha = -2$ and $\beta = -5$.

We know that the standard form of a quadratic polynomial is $p(x) = ax^2 + bx + c$

Upon comparison with the given polynomial, we have
$a = 1,\ b = 7$ and $c = 10$.

Therefore, sum of zeroes, $\alpha + \beta = \frac{-b}{a} = \frac{-7}{1} = -7$
Also, $\alpha + \beta = -2 - 5 = -7$

And, $\alpha\beta = \frac{c}{a} = \frac{10}{1} = 10$
Also, $\alpha\beta = (-2) \times (-5) = 10$
Hence, the relationship between the zeroes and the coefficients is verified.

Question 35

Find the mean of the following data :

Classes	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
Frequency	20	35	52	44	38	31

Solution:

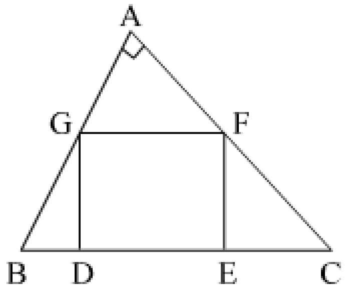
Following is the given data:

Classes	f_i	x_i	$f_i x_i$
0-20	20	10	200
20-40	35	30	1050
40-60	52	50	2600
60-80	44	70	3080
80-100	38	90	3420
100-120	31	110	3410
	220		13,760

Therefore, mean = $\frac{\sum_{i=1}^{6} f_i x_i}{\sum_{i=1}^{6} f_i} = \frac{13760}{220} = 62.55$

Question 36

In the given figure, DEFG is a square in a triangle ABC right angled at A.



Prove that
(i) ΔAGF ~ ΔDBG
(ii) ΔAGF ~ ΔEFC

OR

In an obtuse ΔABC (∠B is obtuse), AD is perpendicular to CB produced. Then prove that $AC^2 = AB^2 + BC^2 + 2BC \times BD$.

Solution:

Given: DEFG is a square inside a triangle ABC right angles at A.
(i) In ΔAGF and Δ DBG,

∠AGF = ∠GBD (Corresponding angles as GF|||BC)
∠GAF = ∠GDB (both right angles)

Therefore, $\Delta AGF \sim \Delta DBG$ (By AA similarity theorem)

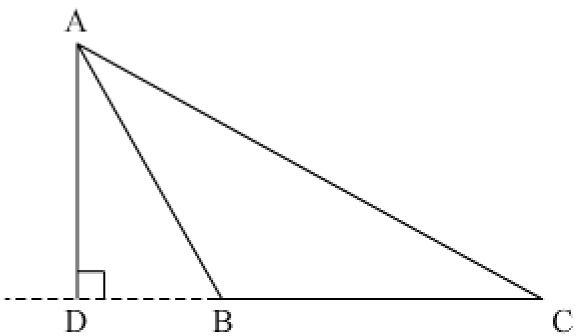
(ii) In ΔAGF and ΔEFC,
$\angle AFG = \angle FCE$ (Corresponding angles as GF||BC)

$\angle GAF = \angle FEC$ (both right angles)

Therefore, $\Delta AGF \sim \Delta EFC$ (By AA similarity theorem)
Hence proved.

OR

Following is the ΔABC, where $\angle B$ is obtuse. AD is perpendicular to CB produced.



In ΔADB, $AB^2 = AD^2 + DB^2$
$\Rightarrow AD^2 = AB^2 - DB^2$ .....(1)
In ΔADC, $AC^2 = AD^2 + DC^2$
$= AB^2 - DB^2 + DC^2$ .....[from (1)]
$= AB^2 + DC^2 - DB^2$
$= AB^2 + (DB + BC)^2 - DB^2$
$= AB^2 + DB^2 + BC^2 + 2\ BC \times BD - DB^2$
$= AB^2 + BC^2 + 2\ BC \times BD$
Hence proved.

Question 37

If 4 times the 4th term of an AP is equal to 18 times the 18th term, then find the 22nd term.

OR

How many terms of the AP : 24, 21, 18, ... must be taken so that their sum is 78?

Solution:

Let the first term be a and the common difference be d.
According to question,
$4a_4 = 18a_{18}$
$\Rightarrow 4(a+3d) = 18(a+17d)$
$\Rightarrow 4a + 12d = 18a + 306d$
$\Rightarrow -14a = 294d$
$\Rightarrow a = -\frac{294}{14}d = -21d$
22nd term = $a_{22} = a + 21d = -21d + 21d = 0$

OR

Given that 24, 21, 18, ... is an AP.
Let n terms should be taken to make the sum 78 then
24 + 21 + 18 + ...to n terms = 78
Here $a = 24,\ d = -3,\ S_n = 78,\ n = ?$
$S_n = \frac{n}{2}\{2a + (n-1)d\}$
$\Rightarrow 78 = \frac{n}{2}\{2 \times 24 + (n-1)(-3)\}$
$\Rightarrow 156 = n(48 - 3n + 3)$
$\Rightarrow 156 = n(51 - 3n)$
$\Rightarrow 3n^2 - 51n + 156 = 0$
$\Rightarrow 3(n^2 - 17n + 52) = 0$
$\Rightarrow (n-13)(n-4) = 0$
$\therefore\ n = 4,\ 13$

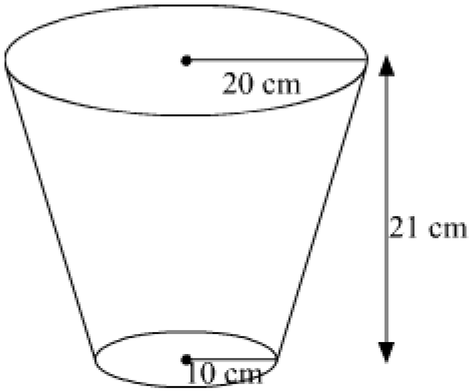
Question 38

An open metal bucket is in the shape of a frustum of cone of height 21 cm with radii of its lower and upper ends are 10 cm and 20 cm respectively. Find the cost of milk which can completely fill the bucket at the rate of ₹ 40 per litre.

OR

A solid is in the shape of a cone surmounted on a hemisphere. The radius of each of them being 3.5 cm and the total height of the solid is 9.5 cm. Find the volume of the solid.

Solution:

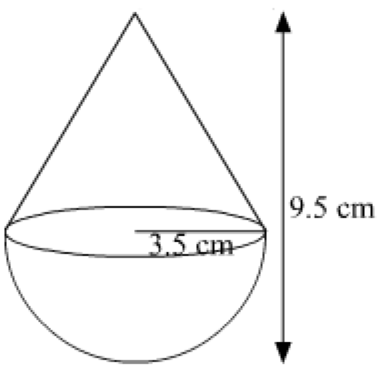


Given, $r_1 = 10\ cm$, $r_2 = 20\ cm$ and $h = 21\ cm$
Therefore, the volume of the bucket (frustum)

$$= \frac{1}{3}\pi h\left(r_1{}^2 + r_2{}^2 + r_1 r_2\right)$$
$$= \frac{1}{3} \times \frac{22}{7} \times 21 \times \left(10^2 + 20^2 + 10 \times 20\right)$$
$$= \frac{1}{3} \times \frac{22}{7} \times 21 \times 700$$
$$= 15400\ cm^3$$
$$= \frac{15400}{1000}\ L$$
$$= 15.4\ L$$

Therefore, the cost of milk $= Rs\ 15.4 \times 40 = Rs\ 616$

OR



Radius of the cone = Radius of the hemisphere = r (say) = 3.5 cm

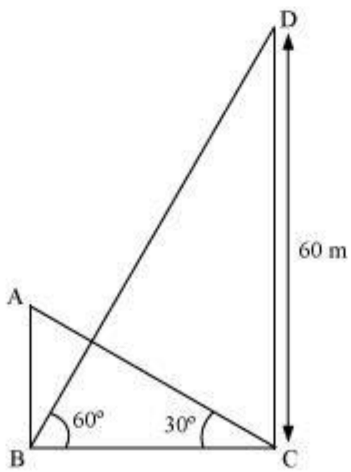
Therefore, height of the cone = (9.5 − 3.5) = 6 cm (as the height of a hemisphere is equal to it's radius)

Therefore, Volume of the solid = Volume of cone + Volume of hemisphere

Question 39

The angle of elevation of the top of a building from the foot of a tower is 30°. The angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 60 m high, find the height of the building.

Solution:



Let AB denote the building and CD denote the tower. The height of the tower is given to be 60 m.
∴ CD = 60 m

We have to find the height of the building i.e. AB.
In ΔBCD,

$$\tan 60° = \frac{DC}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{60\,m}{BC}$$

$$\Rightarrow BC = \frac{60}{\sqrt{3}}\,m \qquad (1)$$

$$\tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{\frac{60}{\sqrt{3}}\,m} \qquad \text{[From (1)]}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{3}\ \text{AB}}{60\ \text{m}}$$

$$\Rightarrow \text{AB} = \frac{1}{\sqrt{3}} \times \frac{60\ \text{m}}{\sqrt{3}} = \frac{60\ \text{m}}{3} = 20\ \text{m}$$

Thus, the height of the building is 20 m.

Question 40

The difference of two natural numbers is 5 and the difference of their reciprocals is 1/10. Find the numbers.

Solution:

Given:

Difference of the two natural numbers = 5

Difference of their reciprocal $= \frac{1}{10}$

Let the two natural numbers be N_1 and N_2.

Therefore,

$N_1 - N_2 = 5$ .....(1)

$\frac{1}{N_2} - \frac{1}{N_1} = \frac{1}{10}$ .....(2)

Considering (2)

$$\frac{1}{N_2} - \frac{1}{N_1} = \frac{1}{10}$$

$$\Rightarrow \frac{N_1 - N_2}{N_1 N_2} = \frac{1}{10}$$

$$\Rightarrow \frac{5}{N_1 N_2} = \frac{1}{10} \qquad \text{[From (1)]}$$

$\Rightarrow N_1 N_2 = 50$ .....(3)

Since, $(a+b)^2 = (a-b)^2 + 4ab$

Therefore,

$$(N_1 + N_2)^2 = (N_1 - N_2)^2 + 4N_1N_2$$

$$\Rightarrow (N_1 + N_2)^2 = 5^2 + 4 \times 50 \qquad \text{[From (1) and (2)]}$$

$$\Rightarrow (N_1 + N_2)^2 = 225$$

$\Rightarrow N_1 + N_2 = 15$ .....(4)

Adding (1) and (4), we get

$2N_1 = 20$

$\Rightarrow N_1 = 10$
Putting the value of N_1 in (4)
$N_2 = 5$
Hence, the natural numbers are 5 and 10.